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Some Remarks on Orthonormal Tetrad Transport†

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Abstract

Some general features of orthonormal tetrad transport along time-like curves are considered while emphasising the kinematical meaning of the 'vierbein' transport process. The issue is further illustrated by comparing Frenet–Serret transport with Fermi–Walker transport.

1. *Introduction*

This paper deals with relativistic kinematics. Some features of the mathematical implements belonging to the 'vierbein' formalism are here discussed in order to point out some useful remarks concerning the transport of orthonormal tetrads along given time-like curves. One of the earliest discussions on the orthonormal 'vierbein' fields in physics is due to Einstein (1928, 1930), when he attempted a unified formulation of gravitation and electromagnetism in his 'distant parallelism' theory. Soon afterwards, the mathematical properties of tetrad fields were elucidated, and the importance of the tetrad formulation for a quantum field theory of gravitation was recognized‡ by Fock (1929) and Rosenfeld (1930). After a period during which the interest in 'vierbeins' subsided, Möller's work on the conservation laws of general relativity (Möller, 1958, 1964) represents a lucid profit of the tetrad field technique, since their use is, in fact, indispensable to obtain the required transformation properties of Einstein's laws of gravitation (Davis & Moss, 1963, 1965). Also, in the interesting attempts to interpret gravitation as a compensating Yang–Mills field, the use of an orthonormal tetrad formalism has been recognised as indispensable (Kibble, 1961). Not long ago, Kaempfer

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‡ See, for example, Dirac (1962).

(1968) showed the usefulness of a 'vierbein' field theory of gravitation for the distinction of genuine gravitational fields from pseudogravitational (i.e. inertial) fields. In summary, we could say that the field theoretic role of the vector tetrad formalism† is well understood today, albeit not widely employed in the current literature.

On the other hand, the use of orthonormal tetrads is not limited to field theoretic considerations; indeed, the formalism has also played a fundamental role in studies of paths in differential geometry (Cartan, 1935). In the present paper we will dwell on these kinematic features of the 'vierbein' technique, for its intuitive value is very appealing. In relativity theory we recognise that the specification of a local frame of reference pertaining to an observer determines a vector tetrad which is being transported along a time-like curve. Thus, for instance, the Fermi–Walker transported tetrads, with their time-like component vector chosen to be tangent to the transporting curve, appear to give us the adequate relativistic generalisation of the Newtonian concept of an accelerated non-rotating frame of reference. Although the concept of 'observer' (as an idealised creature reduced to be a moving point) is perhaps purely rhetorical, it is a useful picturesque notion. Each observer has a world line (time-like, to be sure), representing his history as an ordered continuous sequence of events, and carries a clock (proper time). We obviously normalise the theory by requiring all observers to bear clocks of the same make (standard clocks). However, for the purposes of physics, even if mere kinematics is at issue, these idealised point-observers have to be more richly equipped, otherwise they would be like monads without windows! The world formed by the continuous collection of these isolated beings would have, at most, a purely affine structure not corresponding with the metric structure of the world we live in. Therefore in relativity theory we have to assume, as is done for instance in cosmology, that each observer carries a fundamental equipment composed of the following tools: standard clocks, goniometers, light bulbs, and photocells. (This is in the same spirit as the classical 'rule and compass' platonic requirement of the ancient geometers.) This means that the measurements of time and angles have a locally direct, operationally simple, meaning, while the concept of space distance is a secondary (i.e. derived) construct, which has to be defined somehow by recourse to operations performed by means of the fundamental tools only. What is of interest for us in this context is the fact that these implements enable the local observer to set up a physical frame of reference by choosing three arbitrary linearly independent space-like directions. It must be borne in mind that in making this choice each observer is completely free to use whatever physical criterion he wants to, and that his own history is not affected by this choice. In consequence, corresponding to one and the same time-like curve, we may have quite different local observers (in the sense of local frames of reference), each one spinning relative to the others. Furthermore, it is clear that each of these coinciding point-observers,

† The field theoretic role of tetrad formalism in the framework of general relativity is reviewed by Davis (1966).

with their attached rotating frames, corresponds to a concomitant vector tetrad which is transported along the common world line according to some prescribed equation of motion. All these tetrads have the same time-like component vector, namely, the unit tangent to the transporting curve, and their space-like components represent the three-dimensional spinning frames used by the comoving observers to refer the motion of everything else in the universe.

In the following sections we show how the orthonormal tetrad formalism is indeed the space-time geometric realisation of the kinematic notion of a local, arbitrarily moving, frame of reference. We develop our theme in the general relativistic formalism. In Section 2 we discuss general transport processes which preserve the orthonormality relations among the component vectors of a 'vierbein', emphasising some useful remarks of general validity. In Section 3 we introduce the concept of Frenet-Serret transport as an interesting instance of the issues we are discussing. Finally, in Section 4, we briefly present the well-known Fermi-Walker transport, and compare its behaviour with our Frenet-Serret transport.

2. *Orthonormality Preserving Transport*

The first point we want to emphasise is that there are infinitely many ways of transporting a given tetrad along a curve while preserving the orthonormality relations among the component vectors of the tetrad. Indeed, consider a tetrad† $\{\alpha^{\mu}_{(\nu)}\}$ such that, at some point on a given time-like curve, the relations

$$\alpha^{\mu}_{(\nu)}\alpha^{(\lambda)}_{\mu} = \delta^{(\lambda)}_{(\nu)} \tag{2.1}$$

$$\alpha^{\mu}_{(\nu)}\alpha^{(\nu)}_{\lambda} = \delta^{\mu}_{\lambda} \tag{2.2}$$

hold.‡ Now let us assume the tetrad is transported along the curve while preserving these relations. Then the absolute rate of change of the vectors $\alpha^{\mu}_{(\nu)}$ can be represented in terms of the tetrad itself; namely, we have

$$\dot{\alpha}^{\mu}_{(\nu)} = A_{(\nu)}{}^{(\lambda)}\alpha^{\mu}_{(\lambda)} \tag{2.3}$$

† In this paper we let Greek indices run over the range 0, 1, 2, 3, and Latin indices over 1, 2, 3. We adopt signature (-2) for the metric. For an orthonormal tetrad we write $\{\alpha^{\mu}_{(\nu)}\}$, say, where μ is a tensor index, and (ν) stands for a label denoting the components of the tetrad (we designate with (0) the time-like component, and with (i) the space-like components).

‡ In equations (2.1) and (2.2) we have used the definitions $\alpha^{(\lambda)}_{\mu} = g_{\mu\nu}\eta^{(\lambda)(\rho)}\alpha^{\nu}_{(\rho)}$ where $\eta^{(\lambda)(\rho)} = \eta_{(\lambda)(\rho)} = \text{diag.}\ (1, -1, -1, -1)$ is Minkowski's matrix, and $g_{\mu\nu}$ is space-time covariant metric tensor.

where $\dot{\alpha}^{\mu}_{(\nu)}$ denotes the absolute derivative of $\alpha^{\mu}_{(\nu)}$ with respect to proper time, and the elements $A_{(\nu)}{}^{(\lambda)}$ of the *transport matrix* are scalar functions of proper time. Moreover, since equation (2.1) holds all along the curve, we obtain†

$$A_{(\mu)(\nu)} + A_{(\nu)(\mu)} = 0 \tag{2.4}$$

i.e. the transport matrix is necessarily skew-symmetric. This result is obviously related to the fact that each occurrence of an orthonormal tetrad as it rides on a curve corresponds to rigid rotations in space-time; thus the tetrad evolves from one orientation to the 'next' by means of an infinitesimal Lorentz transformation whose generator is the transport matrix. Furthermore, equations (2.3) and (2.4) together represent a sufficient condition for preserving equations (2.1) and (2.2). That is, provided equations (2.1) (or equation (2.2) for that matter) is given as a set of initial conditions at some point of the curve, the law of motion stated in equation (2.3) together with the skew-symmetry of $A_{(\mu)(\nu)}$ are enough to assure that the orthonormality of the tetrad holds all along the curve. Hence, every linear homogeneous skew-symmetric process of transport preserves orthonormality.

Next, we want to remark that associated with each considered law of transport for orthonormal tetrads we have a corresponding law of transport for vectors and tensors. Considering again equation (2.3) as the given transport law of a 'vierbein' set $\{\alpha^{\mu}_{(\nu)}\}$ we can then write, instead of equation (2.3),

$$\dot{\alpha}^{\mu}_{(\nu)} = A_{\lambda}{}^{\mu}\alpha^{\lambda}_{(\nu)} \tag{2.5}$$

where the transport tensor $A_{\lambda}{}^{\mu}$ of the tetrad is defined as

$$A_{\lambda}{}^{\mu} = A_{(\nu)}{}^{(\sigma)}\alpha^{\mu}_{(\sigma)}\alpha^{(\nu)}_{\lambda} \tag{2.6}$$

Clearly, the transport tensor is skew-symmetric:

$$A_{\mu\nu} + A_{\nu\mu} = 0 \tag{2.7}$$

Then let V^{μ} be a vector propagated along the curve according to the following law:

$$\dot{V}^{\mu} + A^{\mu\nu}V_{\nu} = 0 \tag{2.8}$$

We say that vector V^{μ} is *comoving* with the tetrad along the curve. Clearly, because of the skew-symmetry of the transport tensor of the tetrad, the norm of a comoving vector is a constant of motion along the transporting world line, i.e. only vectors of constant norm on a given curve can be eventually comoving with orthonormal tetrads on that curve. It is clear that the scalar product of two vectors comoving with the same tetrad remains constant along

† Although the labels (ν) have no ordinary tensorial meaning, we can manipulate them confidently as if they were true Lorentz tensorial indices. Thus, in equation (2.4) we have used the definition $A_{(\mu)(\nu)} = A_{(\nu)}{}^{(\lambda)}\eta_{(\lambda)(\nu)}$, as is usually done in this approach. This rule for raising and lowering the labels by means of the Minkowski matrix prevails throughout, and is introduced to secure simplicity.

the curve of transport. In particular, the components of a vector relative to a comoving tetrad are conserved quantities (i.e. proper time independent).

All these properties are well known for vectors undergoing parallel transport or Fermi-Walker transport along time-like world lines. What we want to stress here is the general validity of these statements for every kind of orthonormal transport.

It is also interesting to observe that every given vector of constant norm along a curve can be used to define an *ad hoc* transport tensor on the world line, such that equation (2.8) holds for the given vector. Indeed, given that $(d/d\tau)V_\mu V^\mu = 0$, we define

$$A^{\mu\nu} = (V_\lambda V^\lambda)^{-1}(V^\mu \dot{V}^\nu - V^\nu \dot{V}^\mu) \tag{2.9}$$

This transport tensor is not unique, however. We can always add a skew-symmetric tensor to $A^{\mu\nu}$ in equation (2.9), provided this added tensor is everywhere orthogonal to the vector along the curve. Such a change of 'gauge' does not alter equation (2.8), nevertheless it provides us with a completely different law of transport for tetrads. Therefore, associated with a vector of constant norm along a curve, we have a whole family of 'propagators' $A^{\mu\nu}$ for that curve, of which the simplest member is given in equation (2.9). Conversely, quite different transport processes can be related with one and the same constant-norm vector. This 'gauge' freedom for constructing transport tensor out of a constant-norm vector plays an important role in kinematics, since it corresponds to the fact, alluded to in the Introduction, that the same world line can host different rotating local frames.

3. *Frenet-Serret Transport*

Space-time differential geometry tells us that associated with each event on a world line there is a particularly interesting orthonormal tetrad, the tetrad of Frenet and Serret, which is intimately related with the geometry of the curve. We shall now briefly discuss this 'vierbein', and the corresponding law of transport for vectors, according to the simple ideas presented in the previous section.

We consider a time-like curve. Let u^μ represent the 4-velocity relative to a system of space-time coordinates. In order to introduce the Frenet-Serret tetrad $\{\gamma^\mu_{(\nu)}\}$ at some event $x^\mu(\tau)$ on the curve (τ is proper time) we first define $\gamma^\mu_{(0)} = c^{-1}u^\mu$ (c denotes the velocity of light throughout); we then define, in a progressive manner, three scalars $C_{(i)}$ and three vectors $\gamma^\mu_{(i)}$, $i = 1, 2, 3$, by means of the well-known Frenet-Serret formulas†

$$\dot{\gamma}^\mu_{(\nu)} = C_{(\nu)}{}^{(\lambda)}\gamma^\mu_{(\lambda)} \tag{3.1}$$

† The construction of the Frenet-Serret orthonormal set should be well known to the reader. We only recall here that, while the time-like vector $\gamma^\mu_{(0)}$ is the unit tangent to the curve, the space-like unit vectors $\gamma^\mu_{(i)}$ point in space-time into the three normal directions to the curve, and the scalars $C_{(i)}$ are the corresponding curvatures of the world line.

where the *Frenet-Serret transport matrix* (or curvature matrix) of the world line is given by

$$C_{(\mu)(\nu)} = \begin{bmatrix} 0 & -C_{(1)} & 0 & 0 \\ C_{(1)} & 0 & -C_{(2)} & 0 \\ 0 & C_{(2)} & 0 & -C_{(3)} \\ 0 & 0 & C_{(3)} & 0 \end{bmatrix} \tag{3.2}$$

It is useful to introduce the *rotation matrix* $\omega^{(\mu)(\nu)}$ of the curve, which is defined as the dual of the transport matrix, namely

$$\omega^{(\mu)(\nu)} = \tfrac{1}{2}\epsilon^{(\mu)(\nu)(\lambda)(\rho)}C_{(\lambda)(\rho)} \tag{3.3}$$

where $\epsilon^{(\mu)(\nu)(\lambda)(\rho)}$ is the permutation symbol in four dimensions. Thus, in terms of the rotation matrix, the Frenet-Serret equations become

$$\dot{\gamma}^{\mu}_{(\nu)} = \tfrac{1}{2}\epsilon_{(\nu)(\lambda)(\rho)(\sigma)}\omega^{(\rho)(\sigma)}\gamma^{(\lambda)\mu} \tag{3.4}$$

One easily shows that the vectors of the Frenet-Serret tetrad satisfy the orthonormality relations, cf. equation (2.1) and (2.2), all along the curve.

We now define the corresponding *Frenet-Serret transport tensor* as

$$C^{\lambda\mu} = C_{(\nu)(\rho)}\gamma^{(\nu)\lambda}\gamma^{(\rho)\mu} \tag{3.5}$$

and we say that a vector V^{μ} is being Frenet-Serret transported along the curve if it obeys the law of motion, cf. equation (2.8),

$$\dot{V}^{\mu} + C^{\mu\nu}V_{\nu} = 0 \tag{3.6}$$

which, written explicitly, becomes

$$\begin{aligned} \dot{V}^{\mu} = \{&C_{(1)}(\gamma^{(0)\mu}\gamma^{(1)\nu} - \gamma^{(0)\nu}\gamma^{(1)\mu}) \\ &+ C_{(2)}(\gamma^{(1)\mu}\gamma^{(2)\nu} - \gamma^{(1)\nu}\gamma^{(2)\mu}) \\ &+ C_{(3)}(\gamma^{(2)\mu}\gamma^{(3)\nu} - \gamma^{(2)\nu}\gamma^{(3)\mu})\}\, V_{\nu} \end{aligned} \tag{3.7}$$

From here we obtain the Frenet-Serret tensor of the curve. In order to get a compact and more useful expression for this tensor, we observe that, quite generally, the transport tensor of an orthonormal tetrad is given by

$$A^{\mu\nu} = \alpha^{(\lambda)\mu}\dot{\alpha}^{\nu}_{(\lambda)} \tag{3.8}$$

Thus

$$C^{\mu\nu} = \gamma^{(0)\mu}\dot{\gamma}^{\nu}_{(0)} + \gamma^{(i)\mu}\dot{\gamma}^{\nu}_{(i)} \tag{3.9}$$

But, from equation (3.4), one readily shows

$$\left.\begin{aligned} \dot{\gamma}^{\mu}_{(i)} &= \delta^{(1)}_{(i)}C_{(1)}\gamma^{\mu}_{(0)} + \epsilon_{(i)(j)(k)}\omega^{(k)}\gamma^{(j)\mu} \\ \dot{\gamma}^{\mu}_{(0)} &= C_{(1)}\gamma^{\mu}_{(1)} \end{aligned}\right\} \tag{3.10}$$

where we have defined the scalars

$$\left.\begin{aligned} \omega^{(1)} &= \omega^{(0)(1)} = -C_{(3)} \\ \omega^{(2)} &= \omega^{(0)(2)} = 0 \\ \omega^{(3)} &= \omega^{(0)(3)} = -C_{(2)} \end{aligned}\right\} \tag{3.11}$$

So the following compact expression for the Frenet-Serret transport tensor obtains

$$C^{\mu\nu} = -C_{(1)}(\gamma^{(0)\mu}\gamma^{(1)\nu} - \gamma^{(0)\nu}\gamma^{(1)\mu}) + \epsilon_{(i)(j)(k)}\omega^{(k)}\gamma^{(i)\mu}\gamma^{(j)\nu} \tag{3.12}$$

As for any kind of orthonormal transport, it follows that if we Frenet-Serret transport a 'vierbein' along a time-like curve (not necessarily the Frenet-Serret tetrad!) the 'vierbein' remains orthonormal and its Frenet-Serret components are constants along the curve. But this means that every Frenet-Serret transported tetrad is related to the Frenet-Serret tetrad itself by means of a (constant) Lorentz transformation.

4. *Fermi-Walker Transport*

The well-known geometric process called Fermi-Walker transport allows us to introduce another kind of interesting orthonormal tetrad associated with each event on a time-like world line. According with the general scheme presented in Section 2, one uses the fact that for every time-like curve the 4-velocity is a vector of constant norm, i.e. $u_\mu u^\mu = c^2$, and the Fermi-Walker transport tensor is thus defined as the simplest 'propagator' generated by the 4-velocity, cf. equation (2.9); namely

$$B^{\mu\nu} = c^{-2}(u^\mu g^\nu - u^\nu g^\mu) \tag{4.1}$$

where g^μ is the 4-acceleration. From the first two Frenet-Serret equations, (3.1) and (3.2), one immediately obtains $g^\mu = cC_{(1)}\gamma^\mu_{(1)}$, and so the Fermi-Walker tensor of the curve is given by

$$B^{\mu\nu} = -C_{(1)}(\gamma^{(0)\mu}\gamma^{(1)\nu} - \gamma^{(0)\nu}\gamma^{(1)\mu}) \tag{4.2}$$

Accordingly, a vector V^μ is said to undergo Fermi-Walker transport along the curve if its proper time rate of change is

$$\dot{V}^\mu = C_{(1)}(\gamma^{(0)\mu}\gamma^{(1)\nu} - \gamma^{(0)\nu}\gamma^{(1)\mu})V_\nu \tag{4.3}$$

From this definition it follows that the 4-velocity itself automatically evolves along the curve under Fermi-Walker transport (trivially so, since the 4-velocity was used to generate this type of transport). However, the 4-velocity also undergoes Frenet-Serret transport, as can be easily seen, so that both kinds of processes differ only by a 'gauge' tensor orthogonal to u^μ(cf. equation (3.12) and (4.3)), affording thus an example of the 'gauge' freedom discussed in Section 2.

Compariing equations (3.12) and (4.2), it is interesting to observe that Fermi-Walker and Frenet-Serret become the same transport for time-like curves without torsions, i.e. with vanishing second and third curvatures. These curves correspond physically to one-dimensional motions.

In Section 3 we have defined Frenet–Serret transport while considering *the* Frenet–Serret tetrad (which is a unique structure associated with the world line). In the case of Fermi–Walker transport we have to follow a different logical path since there is no such structure as *the* Fermi–Walker tetrad associated with the curve. But, as for every orthonormal transport, it is

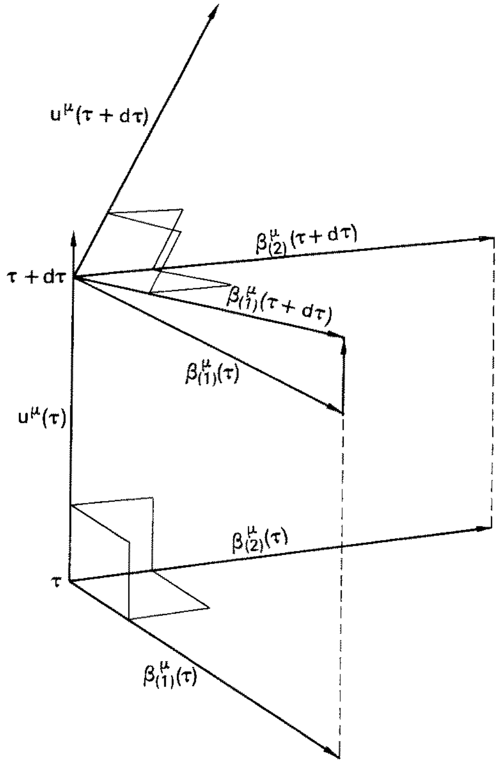

Figure 1.—Space-time diagram showing the evolution of the Fermi–Walker triad $\{\beta^\mu_{(i)};\, i = 1, 2, 3\}$; $\beta^\mu_{(3)}$ has been omitted; cf. equation (4.6).

possible (and interesting) to select the following orthonormal tetrad $\{\beta^\mu_{(\nu)}\}$: At some chosen moment of proper time (τ, say) we take $\beta^\mu_{(\nu)}(\tau) = \gamma^\mu_{(\nu)}(\tau)$ (i.e. this tetrad is initially co-directional with the Frenet–Serret tetrad), and then we let the β-tetrad undergo Fermi–Walker transport along the curve, namely

$$\dot{\beta}^\mu_{(\nu)} + B^{\mu\lambda}\beta_{(\nu)\lambda} = 0 \tag{4.4}$$

Of course, we may concentrate our attention in this particular Fermi–Walker 'vierbein' without loss of generality, since every other Fermi–Walker tetrad will be comoving with this particular one. If we now explicitly write equation (4.4) at proper time τ, we get

$$\dot{\beta}^{\mu}_{(\nu)} = C_{(1)}(\beta^{(0)\mu}\delta^{(1)}_{(\nu)} - \beta^{(1)\mu}\delta^{(0)}_{(\nu)}) \tag{4.5}$$

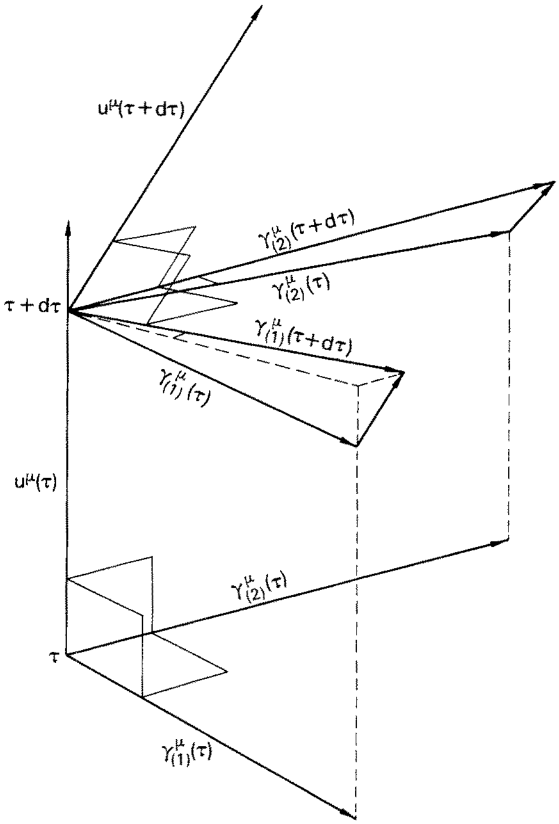

Figure 2.–Space-time diagram showing the evolution of the Frenet-Serret triad $\{\gamma^{\mu}_{(i)}; i = 1, 2, 3\}$; $\gamma^{\mu}_{(i)}$ has been omitted; cf. equation (4.7).

and therefore, at proper time $\tau + d\tau$, to the first order of approximation, we have

$$\beta^{\mu}_{(i)}(\tau + d\tau) \approx \beta^{\mu}_{(i)}(\tau) + d\tau\, \delta^{(1)}_{(i)} C_{(1)}(\tau)\beta^{\mu}_{(0)}(\tau) \tag{4.6}$$

with $i = 1, 2, 3$, representing the evolution of the Fermi-Walker *triad* $\{\beta^{\mu}_{(i)}\}$. On the other hand, for the Frenet-Serret triad $\{\gamma^{\mu}_{(i)}\}$ we obtain, to the same order of approximation,

$$\gamma^{\mu}_{(i)}(\tau + d\tau) \approx \gamma^{\mu}_{(i)}(\tau) + d\tau\, \delta^{(1)}_{(i)} C_{(1)}(\tau)\gamma^{\mu}_{(0)}(\tau) + d\tau\, \epsilon_{(i)(j)(k)}\omega^{(k)}(\tau)\gamma^{(j)\mu}(\tau) \tag{4.7}$$

In Figs. 1 and 2 these results are shown in a sketchy manner, in order to help visualise their meaning. It is easy to see that, while the Frenet-Serret triad is rotating relative to an instantaneous comoving inertial (i.e. free falling) observer, the Fermi–Walker triad evolves without such a rotation. (To this end we introduce an auxiliary inertial tetrad, $\{\alpha^{\mu}_{(\nu)}\}$ say, which is parallel transported along the curve, i.e. $\dot{\alpha}^{\mu}_{(\nu)} = 0$, with initial conditions $\alpha^{\mu}_{(\nu)}(\tau) = \beta^{\mu}_{(\nu)}(\tau) = \gamma^{\mu}_{(\nu)}(\tau)$; we then substitute for $\alpha^{\mu}_{(\nu)}$ in the right-hand members of equations (4.6) and (4.7), and project $\beta^{\mu}_{(i)}(\tau + d\tau)$ and $\delta^{\mu}_{(i)}(\tau + d\tau)$ along the inertial tetrad!)

To end this section let us find the Fermi–Walker transport matrix. According to equation (2.6) we must have

$$B_{(\lambda)(\rho)} = B^{\mu\nu}\beta_{(\lambda)\mu}\beta_{(\rho)\nu} \tag{4.8}$$

that is

$$B_{(\lambda)(\rho)} = C_{(1)}\gamma^{(1)\mu}(\delta^{(0)}_{(\rho)}\beta_{(\lambda)\mu} - \delta^{(0)}_{(\lambda)}\beta_{(\rho)\mu}) \tag{4.9}$$

where $\{\beta^{\mu}_{(\nu)}\}$ is *some* Fermi–Walker transported 'vierbein'. In particular, if at proper time τ we take $\beta^{\mu}_{(\nu)}(\tau) = \gamma^{\mu}_{(\nu)}(\tau)$, we get

$$B_{(\lambda)(\rho)}(\tau) = C_{(1)}(\tau)(\delta^{(0)}_{(\rho)}\delta^{(1)}_{(\lambda)} - \delta^{(1)}_{(\rho)}\delta^{(0)}_{(\lambda)}) \tag{4.10}$$

as we already know from equation (4.5).

5. *Conclusion*

To summarise, we observe that the 'propagators' of the reference tetrads, attached to a given time-like curve, consist of the Fermi–Walker tensor of the curve plus a skew-symmetric tensor orthogonal to the 4-velocity, i.e. a 'gauge' term in the sense of Section 2, representing the rotation of the reference frame relative to an instantaneous comoving free-falling frame. We also observe, quite generally, an enlarged role for Lorentz transformations: Not only inertial tetrads are related by means of them, but accelerated tetrads as well, provided these tetrads are riding on a world line obeying one and the same law of transport. It is clear that, in general, for tetrads riding on a same curve, we go from one 'vierbein' to another by means of (instantaneous) Lorentz transformations, but as proper time elapses these transformations have to be changed, unless the tetrads are comoving. Hence we see that, from the point of view of kinematics, it is comotion, and not inertia (i.e. uniform rectilinear motion), the fundamental property of space-time reflected in the Lorentz group. In this manner we conclude that the formalism of the orthonormal tetrads riding on time-like curves provides all the required tools to deal with arbitrary rotating local frames of reference, with their origins undergoing general time-like motions in space-time.

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